Class Notes - 4

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Dynamic Modelling of Induction Motor:

The derviation of equations for dynamic modeling of induction motor proceeds along following flowchart:

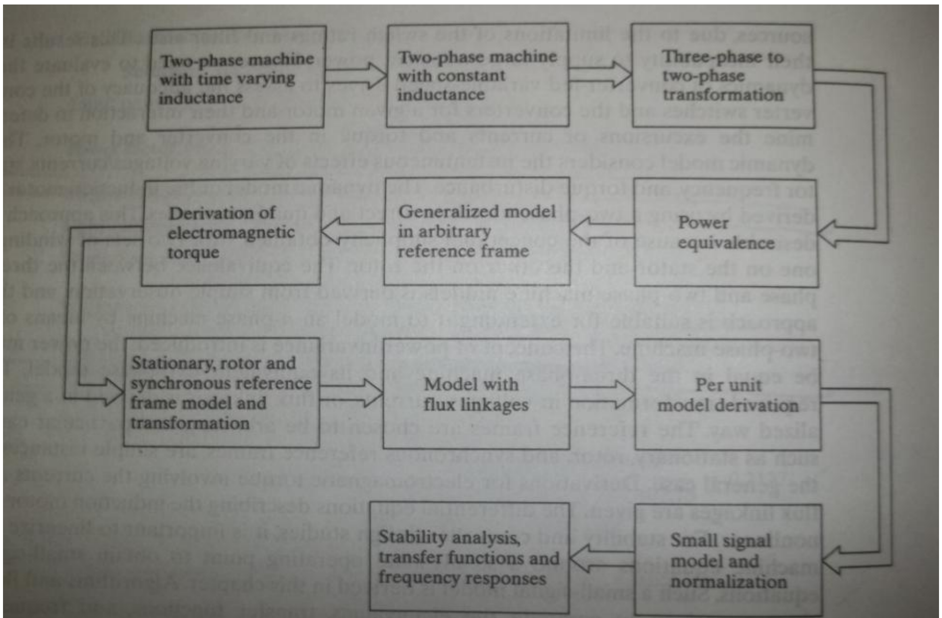

Figure 5.12 Dynamic modeling of the three-phase induction motor

Step -1 : Two Phase Induction Motor

Explanation for why modelling with a 2-phase Induction motor:

Rotating magnetic field is the main component in induction machine, which is generated by stator and induced on the rotor and the interaction between the two results in torque which in turn results in rotation of the rotor.Now to generate a resultant rotating magnetic field in space a minimum of two field 90 degrees out of phase and positioned in quadrature are required.Hence the stator and the rotor rotating magnetic fields can be modeled by two fields each for the rotor and stator respectively.

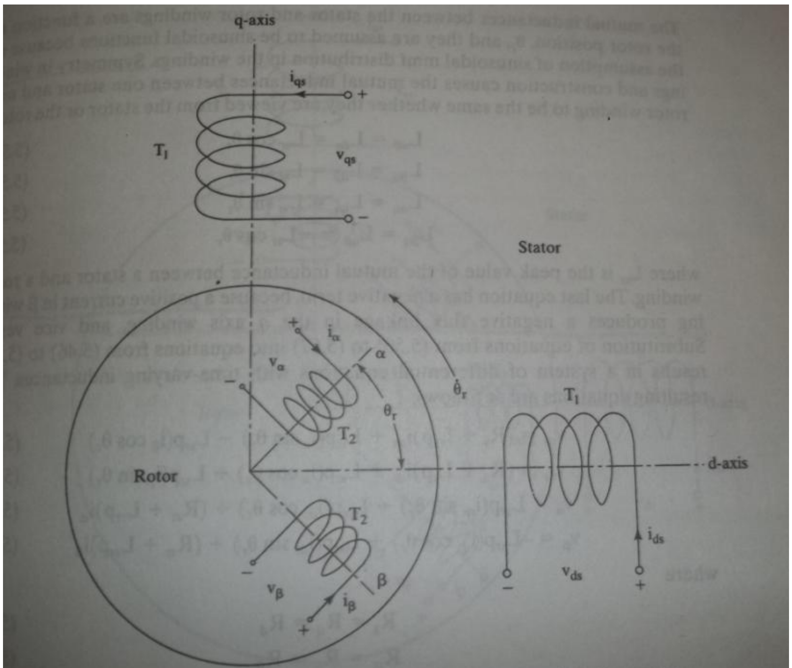

Figure 5.13 Stator and rotor windings of a two-phase induction motor

The KVL equations for the induction motor are as follows:(where p= d/dt)

$$\begin{aligned}
v_{qs} &= R_q i_{qs} + p(L_{qq} i_{qs}) + p(L_{qd} i_{ds}) + p(L_{q\alpha} i_\alpha) + p(L_{q\beta} i_\beta) \\
v_{ds} &= p(L_{dq} i_{qs}) + R_d i_{ds} + p(L_{dd} i_{ds}) + p(L_{d\alpha} i_\alpha) + p(L_{d\beta} i_\beta) \\
v_\alpha &= p(L_{\alpha q} i_{qs}) + p(L_{\alpha d} i_{ds}) + R_\alpha i_\alpha + p(L_{\alpha\alpha} i_\alpha) + p(L_{\alpha\beta} i_\beta) \\
v_\beta &= p(L_{\beta q} i_{qs}) + p(L_{\beta d} i_{ds}) + p(L_{\beta\alpha} i_\alpha) + R_\beta i_\beta + p(L_{\beta\beta} i_\beta)
\end{aligned}$$

But the flux linkages between mutually perpendicular axes is zero.Also assuming the air gap flux to have uniform distribution, the self mutual inductances Lqd = Ldq and so on.

$$\begin{aligned}
v_{qs} &= (R_s + L_s p) i_{qs} + L_{sr} p(i_\alpha \sin\theta_r) - L_{sr} p(i_\beta \cos\theta_r) \\
v_{ds} &= (R_s + L_s p) i_{ds} + L_{sr} p(i_\alpha \cos\theta_r) + L_{sr} p(i_\beta \sin\theta_r) \\
v_\alpha &= L_{sr} p(i_{qs} \sin\theta_r) + L_{sr} p(i_{ds} \cos\theta_r) + (R_{rr} + L_{rr} p) i_\alpha \\
v_\beta &= -L_{sr} p(i_{qs} \cos\theta_r) + L_{sr} p(i_{ds} \sin\theta_r) + (R_{rr} + L_{rr} p) i_\beta
\end{aligned}$$

But if we derive equations for such a motor(see above) ,the flux linkages vary with position and result in equations with varying co-efficients which makes the analysis difficult. Hence we resolve the rotor about fictitious rotor windings placed along with the stator windings. Also we refer the equations to stator to remove the physical isolation between the coils. What we end up is a set of equations having constant co-efficients.

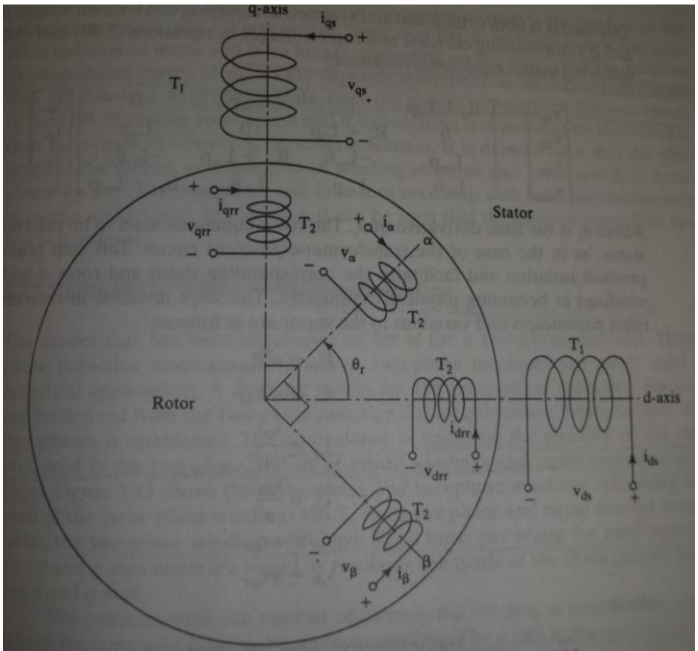

Figure 5.14 Transformation of actual to fictitious rotor variables

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & 0 & L_m p & 0 \\ 0 & R_s + L_s p & 0 & L_m p \\ L_m p & -L_m \dot{\theta}_r & R_r + L_r p & -L_r \dot{\theta}_r \\ L_m \dot{\theta}_r & L_m p & L_r \dot{\theta}_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix}$$

<u>Step 2 : Three Phase to two phase conversion</u>

Two points are to be meet when performing this conversion:

1. The net mmf produced in both the cases must be the same. Hence for the two phase model we need to increase the number turns of windings by 1.5 times.
2. The input power of the three phase motor should be equal to the power input to the two phase machine. This leads to the following condition: $P = V_{abc}^t \, I_{abc} = \frac{3}{2}(v_{qs} i_{qs} + v_{ds} i_{ds})$

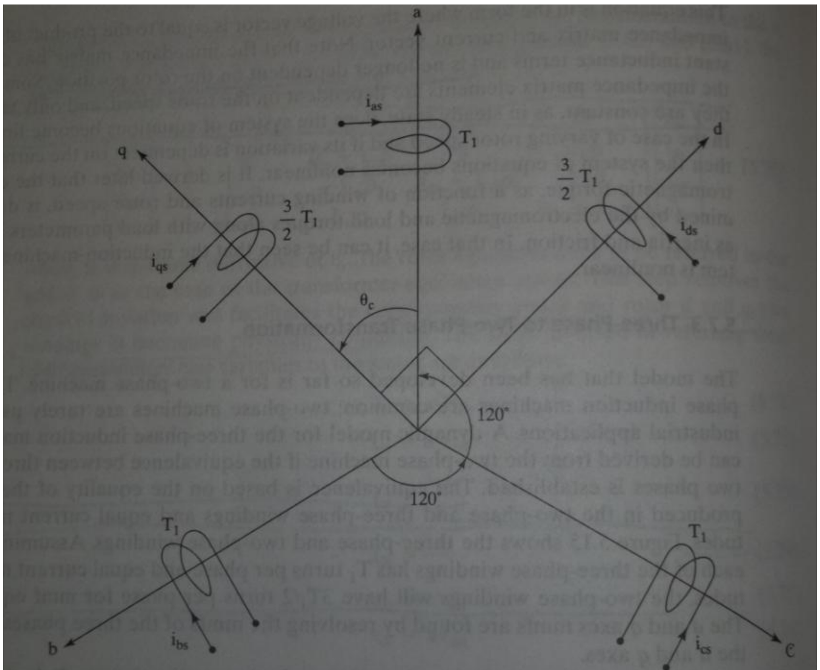

Figure 5.15 Two- and three-phase stator windings

The transformation matrix for 2 phase to 3 phase conversion is as follows(the transformation matrix is applicable to voltages ,currents and flux-linkages):

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_c & \cos\left(\theta_c - \frac{2\pi}{3}\right) & \cos\left(\theta_c + \frac{2\pi}{3}\right) \\ \sin\theta_c & \sin\left(\theta_c - \frac{2\pi}{3}\right) & \sin\left(\theta_c + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Zero sequence current component:

The component i0 is the zero sequence component. It results when the three phase voltages are not balanced. In balanced 3 phase system, the sum f the phase currents is zero and hence i0 is zero.

Model in Arbitrary Rotating Reference Frame:

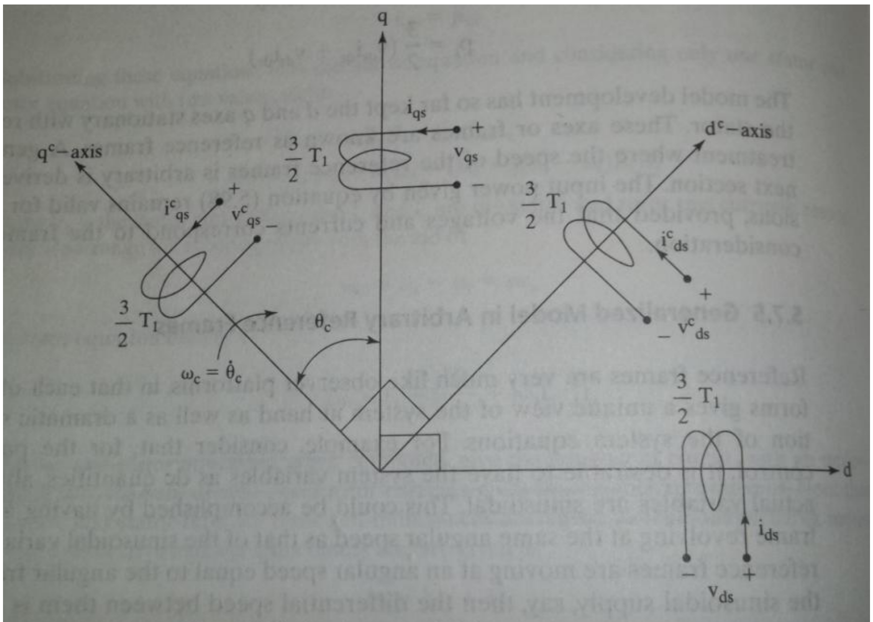

Figure 5.16 Stationary and arbitrary reference frames

Resolving the d-q axes along different rotating reference frames gives us computational advantage depending of the electrical quantity of interest(will be discussed latter).Here the focus is to derive the equations for a frame rotating at arbitrary speed w_c .

Particularly, for the deriving the equation of torque the equations in arbitrary reference frame come handy.

The dq --> dq^c(subscript 'c' denoting arbitrary reference frame) transformation are as follows:

$$i_{qds} = [T^c]\, i^c_{qds}$$

Where ,

$$[T^c] = \begin{bmatrix} \cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix}$$

The impedance matrix thus obtained is:

$$\begin{bmatrix} v^c_{qs} \\ v^c_{ds} \\ v^c_{qr} \\ v^c_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_c - \omega_r) L_m & R_r + L_r p & (\omega_c - \omega_r) L_r \\ -(\omega_c - \omega_r) L_m & L_m p & -(\omega_c - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i^c_{qs} \\ i^c_{ds} \\ i^c_{qr} \\ i^c_{dr} \end{bmatrix}$$

Equation for torque :

We can resolve the above matrix as :

$$V = [R]i + [L]pi + [G]w_r i + [F]w_c i$$

And hence the input power power can be rewritten as:

$$Pi = i^t V = i^t [R] i + i^t [L] pi + i^t [G] w_r i + i^t [F] w_c i$$

Now each of the term has a nice interpretation:

$$i^t [R] i \rightarrow Resitive\ Loses$$

$$i^t [L] pi \rightarrow Rate\ of\ change\ of\ stored\ magnetic\ energy (which\ must\ be\ zero\ in\ steady\ state)$$

*it can be non-zero during dynamic condition.

$$i^t [F] w_c i \rightarrow Reference\ frame\ power\ which\ must\ be\ zero\ as\ the\ frame\ is\ fictitious$$

Hence the Air gap power is what is left i.e.

$$i^t [G] w_r i = T_e\ w_m$$

By expanding we get torque as :

$$T_e\ = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right) Lm\ (i_{qs}^c i_{dr}^c - i_{ds}^c i_{qr}^c)$$

Similarity between this equation and torque equation of DC motor:

For DC motor : $T_e = Kf\ If\ Ia$

$where\ Kf\ is\ a\ function\ of\ mutual\ inductance\ between\ stator\ and\ the\ rotor\ winding$.

In induction motor too we have torque to be function of L_m, the mutual inductance between the rotor and the stator windings.

In case of DC motor ,by virtue of the construction of the motor , $If(\emptyset f)$ and $Ia(\emptyset a)$ are perpendicular to each other.

In case of induction motor, when modeled using d-q axes in arbitrary reference frame, we too have torque to be dimensionally equal to products of $i_{qs}^c i_{dr}^c$ and $i_{ds}^c i_{qr}^c$ which are individually composed of mutually perpendicular d and q components of stator and rotor currents respectively.

Commonly used rotating reference frame: (Which to use where?)

We have 3 commonly used reference frames each of which offer different advantages depending on which is our quantity of interest.

Stator Reference frame: ($w_c = 0$)

This frame is used when stator variables are required to be actual and rotor variables can be fictitious. Eg. for stator controlled induction motor.

Rotor Reference frame: ($w_c = w_r$)

Similar to the stator reference frame, this frame is used when we are interested in rotor variables (wound rotor machine) .Example in slip power recovery system.

Synchronous Reference frame: $(w_c = w_s)$

The unique advantage of this frame is that the sinusoidal variables get converted into DC quantities in synchronous frame. This model is useful in evaluating steady state values of variables which is further used in small-signal modeling.

Equation in flux linkages

Instead of current variables, flux variables can be used in modeling of induction motor equations.

$$\left.\begin{aligned}
\lambda_{qs}^{c} &= L_s i_{qs}^{c} + L_m i_{qr}^{c} \\
\lambda_{ds}^{c} &= L_s i_{ds}^{c} + L_m i_{dr}^{c} \\
\lambda_{qr}^{c} &= L_r i_{qr}^{c} + L_m i_{qs}^{c} \\
\lambda_{dr}^{c} &= L_r i_{dr}^{c} + L_m i_{ds}^{c}
\end{aligned}\right\}$$

It offers two advantages:

1. The number of variables is reduced.
2. Flux linkages are continuous even when voltages and current are discontinuous.(Could find any explanation for this point)

Both the above points lead to computational efficiency and numerical stability.

Space –Phasor Philosophy:

Instead of representing stator and rotor flux linkages by d-q axes, we can represent it by a single phase, one each for rotor and stator .

It results in further reducing the number on equations .Now since we have just two windings it can be more closely related to DC motor with a motive to try to decouple the torque and flux .