Class Notes -3

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Generic Steps followed in Closed Loop Control of DC motor :

Need for Controller:

1. Better response
2. To operate the motor within rated limits of current and speed(protection)
3. Better Steady state accuracy

Variable that be controlled in a motor drive:

1. Position
2. Speed
3. Torque

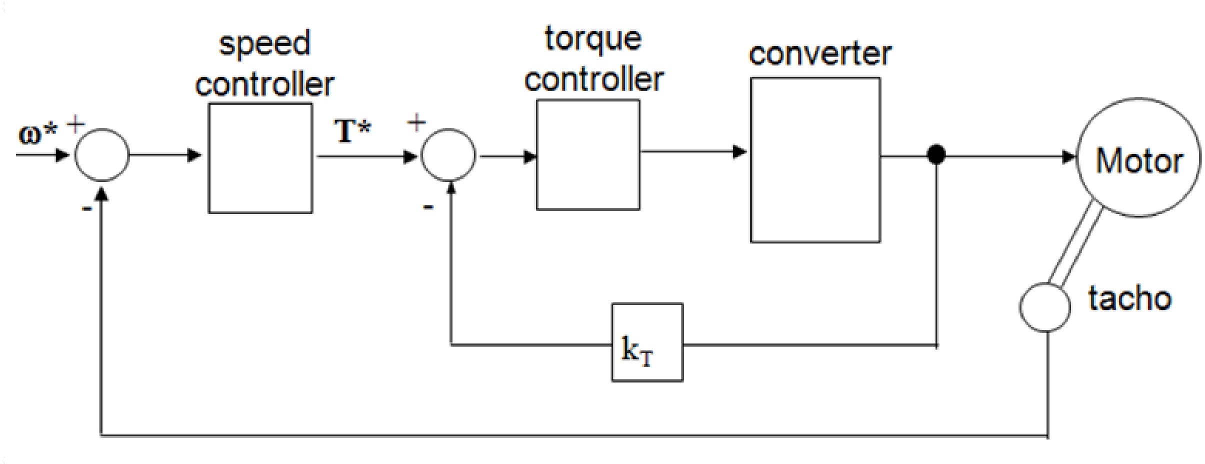
The following discusses the design of speed controller for a DC motor drive which proceeds as:

1. Modeling of the motor and the measurement peripherals and convertor

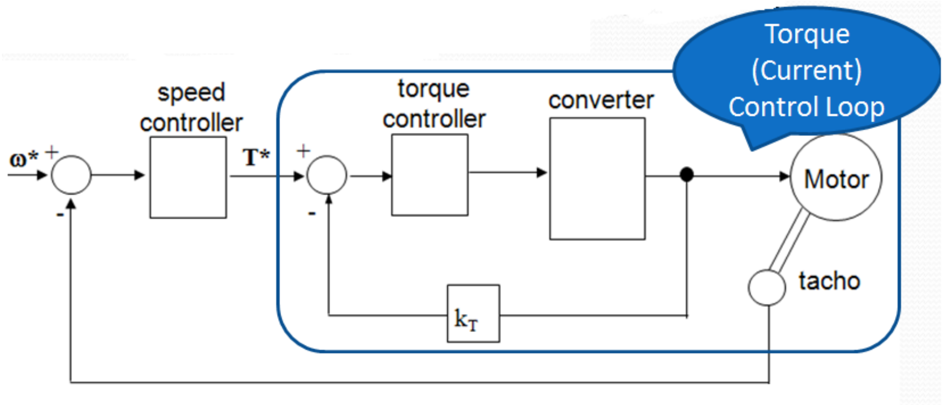
2. Simplification of the model for analytical calculations

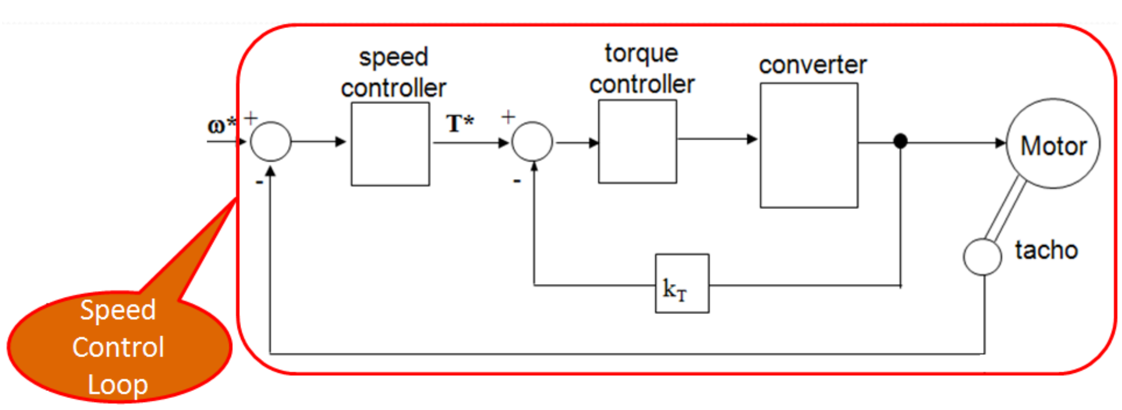
3. Design of PI speed and current controllers for the simplified model.

Block Diagram for Closed loop speed control of DC motor:

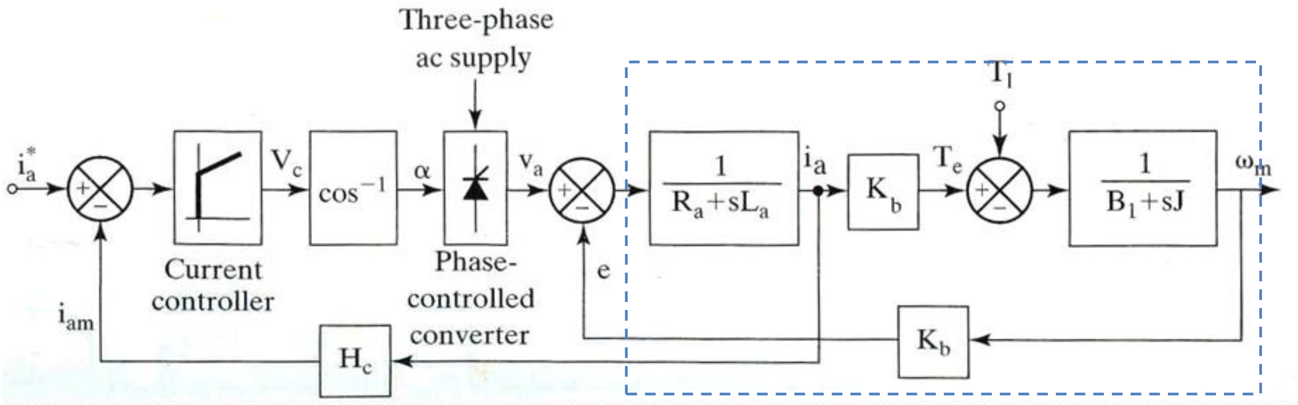


The control structure consists of a cascade control. This is generally adopted when the dynamics of the inner loop are faster than the dynamics of the outer loop. Here we have two loops the current (torque) loop and the speed loop. Approximately, the inner loop time constant is dominated by Te (Electrical time constant) and the outer loop response by Tm (mechanical time constant) ,and generally Tm>>Te. Thus having a cascade structure can lead to performance improvement in closed loop control of DC motor.





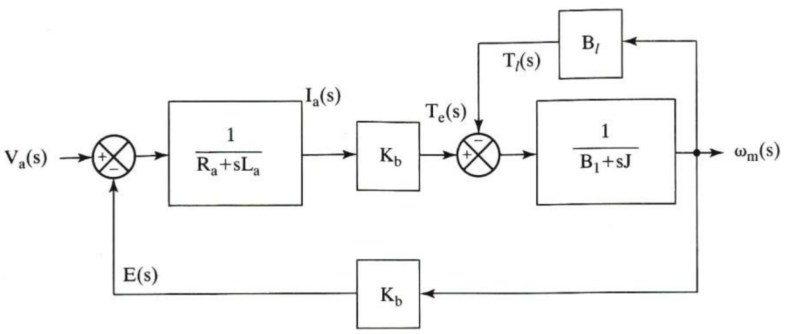
The following proceeds with deriving the transfer functions of the individual blocks of the motor and then developing analytical method to determine the controller constants.

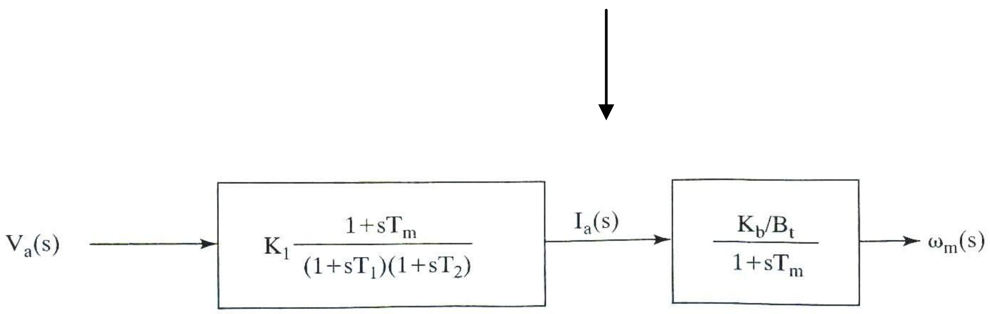


Va to wm transfer function for the motor:

Simplying the transfer function to simplify the inherent inner emf loop :

$$\frac{\omega_m(s)}{V_a(s)} = \frac{\omega_m(s)}{I_a(s)} \cdot \frac{I_a(s)}{V_a(s)} \longrightarrow \boxed{\frac{\omega_m(s)}{I_a(s)} = \frac{K_b}{B_t(1+sT_m)}} \longrightarrow \boxed{\frac{I_a(s)}{V_a(s)} = K_1 \frac{(1+sT_m)}{(1+sT_1)(1+sT_2)}}$$





Where the constants are :

$$T_m = \frac{J}{B_t} \qquad \boxed{K_1 = \frac{B_t}{K_b^2 + R_a B_t}}$$

$$B_t = B_1 + B_L \qquad \boxed{-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left(\frac{R_a}{L_a} + \frac{B_t}{J}\right) \pm \sqrt{\frac{1}{4}\left(\frac{R_a}{L_a} + \frac{B_t}{J}\right)^2 - \left(\frac{R_a B_t}{J L_a} + \frac{K_b^2}{J L_a}\right)}}$$

Model for the convertor:

The 3 phase controlled rectifier output cannot be changed until the next triggering pulse. Thus there is a delay introduced by the convertor. Also the output of the rectifier is proportional to the cosine of the triggering angle thus leading to a non-linear relationship between input and output. For linear controller design we use cosine-wave crossing method to linearize this function.

$$\alpha = \cos^{-1}\left(\frac{v_c}{V_{cm}}\right)$$

$$V_{dc} = \frac{3}{\pi}V_{\mathrm{L-L},m}\cos\alpha = \frac{3}{\pi}V_{\mathrm{L-L},m}\cos\left(\cos^{-1}\frac{v_c}{V_{cm}}\right) = \frac{3}{\pi}\frac{V_{\mathrm{L-L},m}}{V_{cm}}v_c = K_r v_c$$

The delay which depends on the supply frequency is given by:

$$T_r = \frac{1}{2}\times\frac{60}{360}\times\frac{1}{f_s} = \frac{1}{12}\times\frac{1}{f_s}$$

$$\boxed{\mathrm{G_r(s)} = \frac{K_r}{(1+sT_r)}}$$

Current Feedback:

It is modeled by a transfer constant transfer function H_c .Or if filtering is required low pass filter is included.

Speed Feedback:

The speed feedback depends on the tachometer response time which does generally have delay. It is modeled as follows: (where Kw and Tw are gain and time constant respectively)

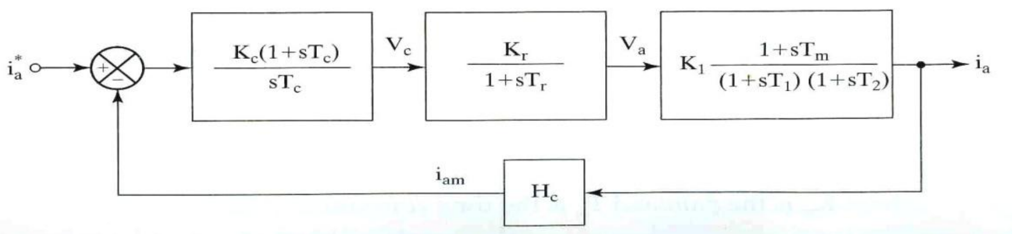
$$\mathrm{G_\omega(s)} = \frac{K_\omega}{(1+sT_\omega)}$$

Design of the controller:

As discussed before design proceeds from the innermost loop to the outer loop.

Current Controller:

To have good transient performance and zero steady state error the controller is implemented is a PI controller.



So aim in the controller design is to find the controller constants Kc and Tc for good transient and steady state performance. To find these constants analytically the order of the system needs to be reduced for which appropriate approximations and pole-zero cancellation comes in handy.

Open loop gain function:

$$\mathrm{GH_{ol}}(s) = \left\{\frac{K_1 K_c K_r H_c}{T_c}\right\} \frac{(1+sT_c)(1+sT_m)}{s(1+sT_1)(1+sT_2)(1+sT_r)}$$

In short we will reduce the open loop system from 4th → 3rd →2nd order for which we have standard procedures to decide on Kc and Tc. Firstly we set the controller time constant t be equal to T2.ie. $\boxed{T_c = T_2}$

and assume that

$\boxed{(1+sT_m) \cong sT_m}$ using this approximation with the above setting, results in pole-zero cancellation.

after which the final open loop transfer function which we get is:

$$\mathrm{GH_{ol}}(s) \cong \frac{K}{(1+sT_1)(1+sT_r)}$$

for which the closed loop transfer function's characteristic equation comes out to be:

$\boxed{(1+sT_1)(1+sT_r)+K}$ ⟶ $\boxed{T_1T_r\left\{s^2 + s\left(\frac{T_1+T_r}{T_1T_r}\right) + \frac{K+1}{T_1T_r}\right\}}$

Now we can compare this with standard

2nd order equation $s^2 + 2\zeta\omega_n s + \omega_n^2$ and decided upon Zetha(damping factor) and calculate wn and thereby find Kc.

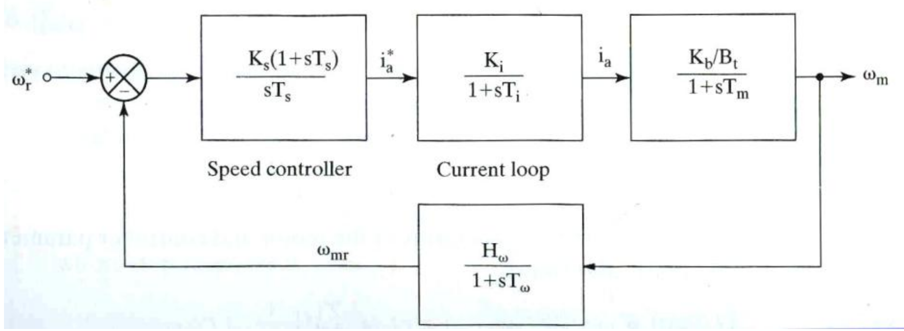
Speed Controller :

Using the speed controller thus designed, we design the speed controller.

But if we were to use the second order apprx. Model of current controller, the analysis still would be on 4th order.Hence for analytical calculation purposes, we approximate the current loop model to first order one as

$\frac{K_i}{(1+sT_i)}$ where $\boxed{T_i = \frac{T_3}{1+K_{fi}}}$ $\boxed{K_i = \frac{K_{fi}}{H_c}\frac{1}{(1+K_{fi})}}$ $\boxed{K_{fi} = \frac{K_1 K_c K_r H_c T_m}{T_c}}$

And thus the speed controller loop becomes:



If we consider the speed feedback transfer function to be unity, we get simplified open loop transfer function of order 3. But for calculation purposes we need to reduce it to standard second order function(using pole zero cancellation method) for which the speed time constant is designed equal to Tm.i.e. $\boxed{T_s = T_m}$

Hence we get the open loop transfer function as:

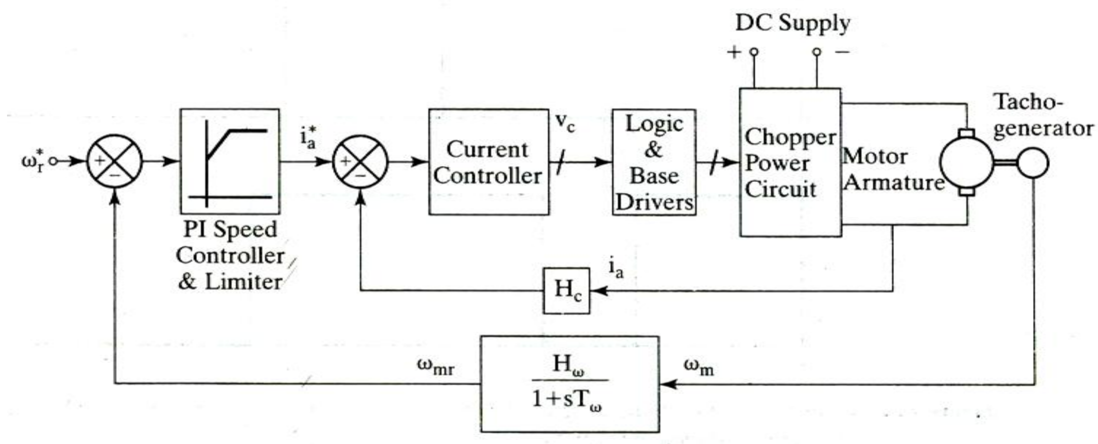
$$GH(s) \cong \frac{K_\omega}{s(1+sT_i)} \quad \text{where} \quad \boxed{K_\omega = \frac{K_B K_s K_i}{B_t T_s}}$$

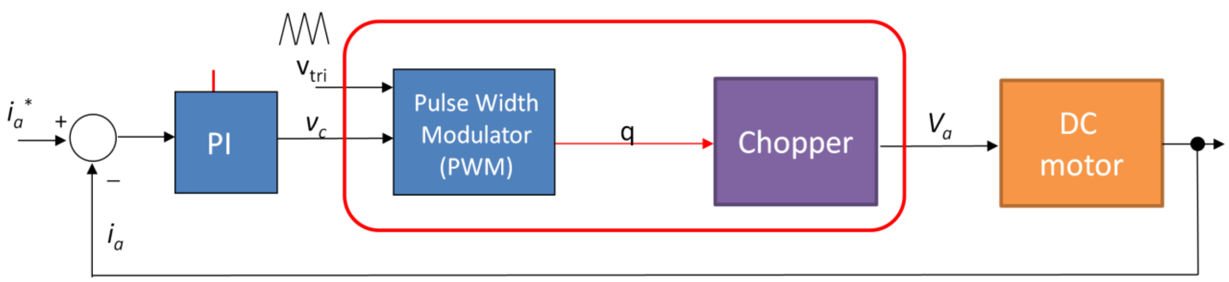
And the closed loop character equation becomes:

$$\boxed{s(1+sT_i)+K_\omega} \quad \text{which is equal to} \quad \boxed{T_i\left\{s^2 + s\left(\frac{1}{T_i}\right)+\frac{K_\omega}{T_i}\right\}}$$

Now the determination of speed gain constant can proceed as previously using standard method : fixing zetha(damping factor) →finding wn →finding Kc.

<u>Controller design when using Chopper DC-DC convertor:</u>





The controller transfer function too is given by:

$$\boxed{G_r(s) = \frac{K_r}{(1+sT_r)}} \quad \text{where} \quad \boxed{K_r = \frac{V_a}{v_c} = \frac{V_{dc}}{2V_{tri}}} \quad \text{(for 2- Quadrant Chopper) and} \quad \boxed{T_r = \frac{1}{2f_c}}$$

The rest analysis proceeds the same as with the case of phase controller rectifier done previously.